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THESIS

Implementation of a Reliability Shorthand
on the TI-59 Handheld Calculator

by

Hans-Eberhard Peters

October 1982

Thesis Advisor:

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T205425

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Implementation of a Reliability Short- hand on the TI-59 Handheld Calculator		5. TYPE OF REPORT & PERIOD COVERED Master' Thesis; October 1982
7. AUTHOR(s) Hans-Eberhard Peters		6. PERFORMING ORG. REPORT NUMBER
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey, California 93940		8. CONTRACT OR GRANT NUMBER(s)
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Postgraduate School Monterey, California 93940		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) Naval Postgraduate School Monterey, California 93940		12. REPORT DATE October 1982
		13. NUMBER OF PAGES 72
		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) TI-59 Handheld Calculator Programmable Calculator Reliability Shorthand		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) It is shown how a reliability shorthand can be implemented on a handheld calculator. Assuming constant failure rates, basic structures are used to show how the shorthand can be applied. Several examples are worked out that show, how, with component failure rates as input, a handheld calculator can be		

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Two TI-59 programs are provided as a computational aid.

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Implementation of a Reliability Shorthand
on the TI-59 Handheld Calculator

by

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Submitted in partial fulfillment of the
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MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the
NAVAL POSTGRADUATE SCHOOL
October, 1982

ABSTRACT

It is shown how a reliability shorthand can be implemented on a handheld calculator.

Assuming constant failure rates, basic structures are used to show how the shorthand can be applied. Several examples are worked out that show, how, with component failure rates as input, a handheld calculator can be used to compute the reliability of a system.

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I. INTRODUCTION

Systems and components can be in either of two states: either they are functioning or they have failed. The ability, that a system stays functioning over a predetermined time interval is called its reliability. It is generally not realistic to assume that a system, say a lightbulb, will fail at a specified time, but rather that T , the time to failure, is a random variable which has a probability distribution that can be specified. The probability distribution for a time to failure is called its life distribution. In this paper we will solely be concerned with one specific type of life distribution which is especially important in reliability theory and practice, the exponential distribution. It has the property that the remaining life of a used component is independent of its age (the "memoryless" property), i.e. a functioning component is always as good as new, the failure rate is constant. The memoryless property is the basis for a reliability shorthand, one that can be implemented on a handheld calculator.

Depending on the size, structure and life distribution of a system, probability statements about its time to

failure are in general not easily achieved. Forming the sum of independent life lengths (i.e. convolving the corresponding life distributions) requires knowledge of integral calculus and computations can become rather tedious.

In the case of the exponential distribution, though, computations can be simplified by translating the problem into a simple shorthand notation and using this shorthand as input for some computing device.

In this paper we will show how a reliability shorthand can be implemented on a handheld calculator. Basic structures are used to show how the shorthand can be applied. Two TI-59 programs are provided as a computational aid. Formulas for the convolution of up to four exponential random variables can be found in Appendix A. Appendix B contains a user guide to the TI-59 programs.

II. THE CONCEPT OF A RELIABILITY SHORTHAND

A. BASIC NOTATION

The survival function of a life length can be derived from the distribution function.

Let

T : life length

$F(t) = P(T \leq t)$ be the distribution function of T

Then

$$\begin{aligned}\bar{F}(t) &= P(T > t) \\ &= 1 - F(t)\end{aligned}$$

is the survival function of T .

In the case of the exponential distribution, $\bar{F}(t) = e^{-\lambda t}$, where λ is the failure rate. Translated into shorthand, the life distribution is denoted

$$\text{EXP}(\lambda).$$

B. CONVOLUTION OF DISTRIBUTIONS

When independent random lives are summed up, the corresponding life distributions have to be convolved to determine the probability that the sum of the lives will exceed a specified time t . Let

T_1, T_2 : independent life lengths

$\bar{F}_1(t), \bar{F}_2(t)$: the corresponding survival functions

$f_1(t), f_2(t)$: the corresponding density functions

$T = T_1 + T_2$: the total life length

Then

$$\begin{aligned}\bar{F}(t) &= P(T > t) \\ &= P(T_1 + T_2 > t) \\ &= \bar{F}_1(t) + \int_0^t \bar{F}_2(t-s) f_1(s) ds.\end{aligned}$$

This means that T will exceed a specified time t when

-either T_1 exceeds t

-or T_1 is smaller than t , say equal to s , and T_2 exceeds $t-s$.

Integration with respect to s (i.e. summing over all possible values of s) is called the convolution of F_1 and T_2 .

When T_1 and T_2 are both exponentially distributed with failure rates λ_1 and λ_2 , i.e.

$$\begin{aligned}\bar{F}_1(t) &= e^{-\lambda_1 t} \\ \bar{F}_2(t) &= e^{-\lambda_2 t},\end{aligned}$$

then the survival function of T is

$$\bar{F}(t) = e^{-\lambda_1 t} + \int_0^t e^{-\lambda_2(t-s)} \lambda_1 e^{-\lambda_1 s} ds.$$

Translated into shorthand, the survival function is denoted

$$\text{EXP}(\lambda_1) + \text{EXP}(\lambda_2).$$

This shorthand notation is heuristically apparent. We can visualize a 1 component / 1 spare system with $\text{Exp}(\lambda_1)$ and $\text{Exp}(\lambda_2)$ lives respectively. From component 1 the system has an $\text{Exp}(\lambda_1)$ life to begin with. When component 1 fails, the system has an extra $\text{Exp}(\lambda_2)$ life.

C. MIXTURE OF DISTRIBUTIONS

1. MIX-Notation

In the previous chapter, we formed the sum of independent random lives, which each had weight one, i.e.

$$T = T_1 + T_2.$$

Now consider

$$T = \begin{cases} T_1 & \text{with probability } p_1 \\ T_2 & \text{with probability } p_2 \end{cases}$$

where $p_1 + p_2 = 1$.

Let D_1 and D_2 be the probability distributions of the random variables T_1 and T_2 respectively. The corresponding survival functions are $\bar{F}_1(t)$ and $\bar{F}_2(t)$.

Then

$$\bar{F}(t) = p_1 \bar{F}_1(t) + p_2 \bar{F}_2(t).$$

In shorthand, the mixture of distributions D_1 and D_2 with respect to the mixing probabilities p_1 and p_2 is denoted

$$\text{MIX} [p_1 D_1 , p_2 D_2].$$

2. Distributive Law

Now let

$$T = T_3 + T'$$

where

$$T' = \begin{cases} T_1 & \text{with probability } p \\ T_2 & \text{with probability } 1-p. \end{cases}$$

Then

$$T = T_3 + \begin{cases} T_1 & \text{with probability } p \\ T_2 & \text{with probability } 1-p. \end{cases}$$
$$T = \begin{cases} T_3 + T_1 & \text{with probability } p \\ T_3 + T_2 & \text{with probability } 1-p. \end{cases}$$

The distributive law holds due to the fact that the sum of the mixing probabilities for T_1 and T_2 is equal to one.

The survival function of T can be found by convolution:

$$\bar{F}(t) = \bar{F}_3(t) + \int_0^t (p\bar{F}_1(t-s) + (1-p)\bar{F}_2(t-s))f_3(s)ds.$$

With D_1, D_2, D_3 being the probability distributions for T_1, T_2, T_3 , the distributive law can be applied to the shorthand notation:

$$D_3 + \text{MIX} [pD_1, (1-p)D_2] = \text{MIX} [p(D_1 + D_3), (1-p)(D_2 + D_3)].$$

Graphically this can be represented as follows:

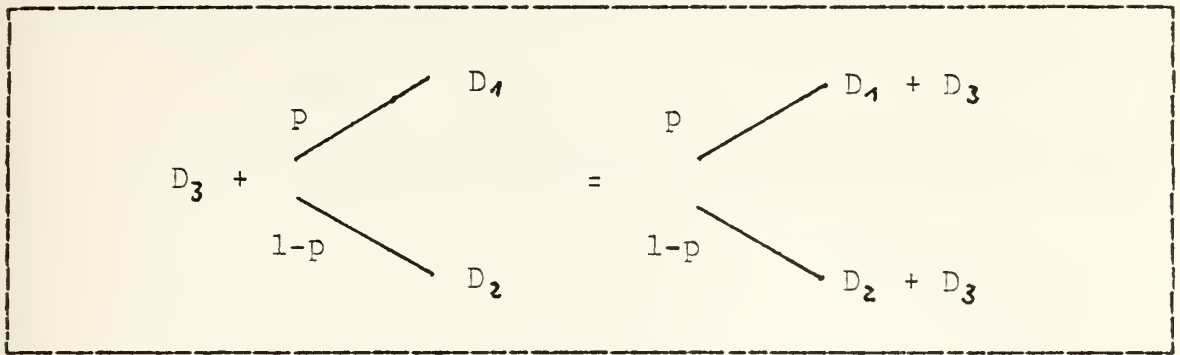


Figure 1: Distributive Property of the MIX-Notation

3. Degeneracy at the Origin

Let

$$P(T=0) = 1.$$

Then the distribution of T is degenerate at zero.

In shorthand notation, such a distribution is called the ZERO-distribution.

Now let $T = T_1 + T_0$

where T_1 and T_0 have probability distributions D_1 and ZERO and survival functions $\bar{F}_1(t)$ and $\bar{F}_0(t)$ respectively.

Then

$$\begin{aligned}\bar{F}(t) &= \bar{F}_1(t) + \int_0^t \bar{F}_0(t-s) f_1(s) ds \\ &= \bar{F}_1(t).\end{aligned}$$

The ZERO-distribution doesn't add anything to another distribution, so for instance

$$D_1 + \text{ZERO} = D_1$$

$$D_2 + \text{MIX}[pD_1, (1-p)\text{ZERO}] = \text{MIX}[p(D_1 + D_2), (1-p)D_2].$$

III. APPLYING A RELIABILITY SHORTHAND

After this brief survey over the concept of a reliability shorthand we will now show how the shorthand can be applied. To do so we will use basic structures. Part A of this chapter will give examples whose representation in shorthand requires only basic notation described in Chapter II, Parts A and B, whereas Part B of this chapter will give examples whose representation in shorthand makes use of the MIX-notation and the ZERO-distribution.

A. SUMS OF EXPONENTIALS WITH WEIGHT ONE

1. Simple Series System

A series system is a system which is functioning, when all its components are functioning. A two-component series system can be graphically represented as shown in Fig.2 .

Let

T : life of the system

T_1 : life of component 1

T_2 : life of component 2

$$\begin{aligned}\bar{F}_1(t) &= \text{survival function of component 1} \\ &= e^{-\lambda_1 t}\end{aligned}$$

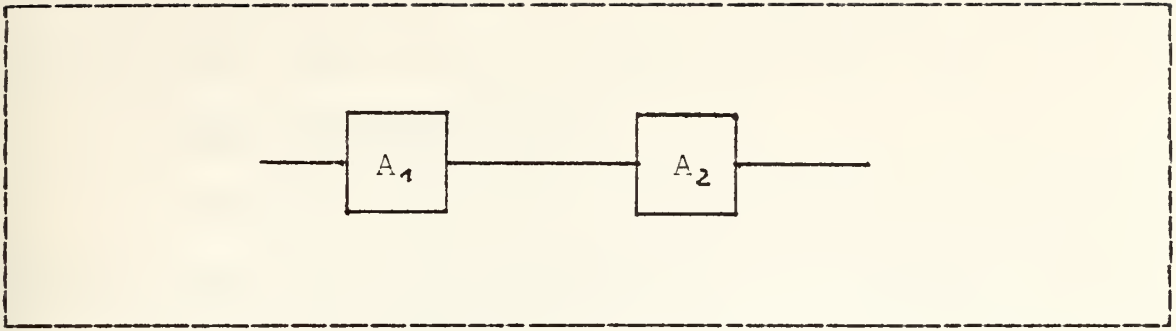


Figure 2: Two-Component Series System

$$\begin{aligned}\bar{F}_2(t) &= \text{survival function of component 2} \\ &= e^{-\lambda_2 t}.\end{aligned}$$

Then

$$T = \min(T_1, T_2)$$

$$\begin{aligned}\bar{F}(t) &= \text{survival function of the system} \\ &= P(\min(T_1, T_2) > t) \\ &= P(T_1 > t, T_2 > t)\end{aligned}$$

Assuming independence of the two components

$$\begin{aligned}\bar{F}(t) &= P(T_1 > t) P(T_2 > t) \\ &= \bar{F}_1(t) \bar{F}_2(t) \\ &= e^{-\lambda_1 t} e^{-\lambda_2 t} \\ &= e^{-(\lambda_1 + \lambda_2)t}.\end{aligned}$$

The shorthand notation for this system is

$$\text{EXP}(\lambda_1 + \lambda_2).$$

This is intuitively apparent, as the system has an exponential survival function with failure rate $\lambda_1 + \lambda_2$.

2. Simple Parallel System

A parallel system is a system which is functioning, when at least one of its components is functioning. A two-component parallel system can be graphically represented as follows:

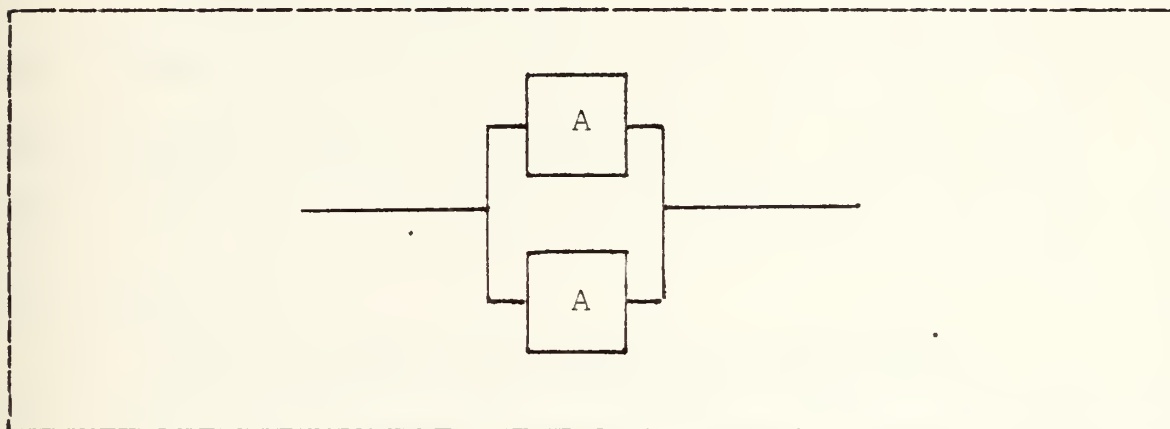


Figure 3: Two-Component Parallel System

Let

$$T_1 \sim \text{EXP}(\lambda) , T_2 \sim \text{EXP}(\lambda) .$$

Then

$$T = \max(T_1, T_2)$$

$$\bar{F}(t) = P(\max(T_1, T_2) > t)$$

$$= 1 - P(\max(T_1, T_2) \leq t)$$

$$= 1 - P(T_1 \leq t, T_2 \leq t)$$

Assuming independence of the two components,

$$\begin{aligned}
 \bar{F}(t) &= 1 - P(T_1 \leq t) P(T_2 \leq t) \\
 &= 1 - F_1(t) F_2(t) \\
 &= 1 - (1 - e^{-\lambda t}) (1 - e^{-\lambda t}) \\
 &= 1 - (1 - 2e^{-\lambda t} + e^{-2\lambda t}) \\
 &= 2e^{-\lambda t} - e^{-2\lambda t}.
 \end{aligned}$$

The shorthand notation for the system is

$$\text{EXP}(2\lambda) + \text{EXP}(\lambda).$$

This follows intuition as the system has an $\text{EXP}(2\lambda)$ life to begin with and when one component fails it has an extra $\text{EXP}(\lambda)$ life due to the memoryless property of the exponential distribution.

3. Standby-System with Dissimilar Components

Suppose a system consists of two components, one active and one spare. The active component stays in service until it fails and then immediately is replaced by the spare.

Let the time to failure of the two components be $T_1 \sim \text{EXP}(\lambda_1)$ and $T_2 \sim \text{EXP}(\lambda_2)$ respectively. Then the system time to failure is

$$T = T_1 + T_2$$

and the survival function of the system is

$$\bar{F}(t) = P(T > t)$$

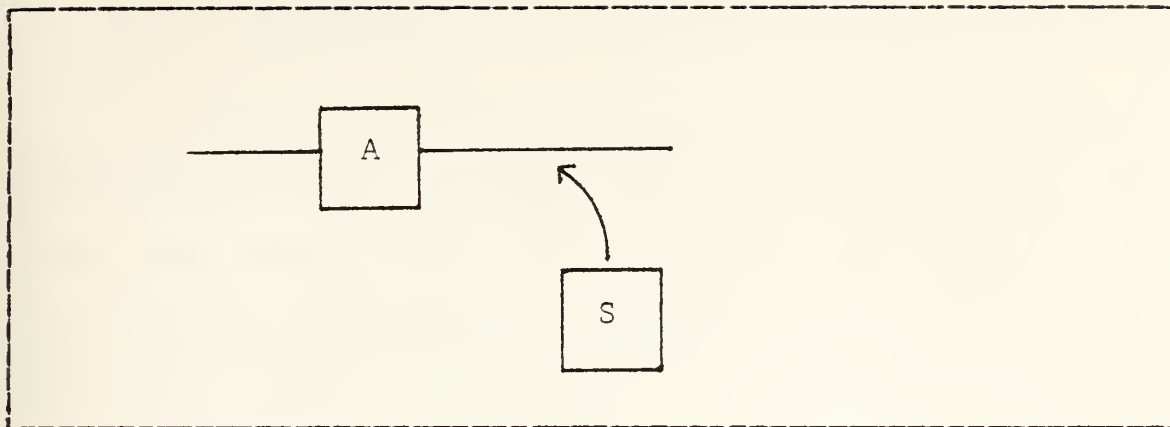


Figure 4: Standby System

$$\begin{aligned}
 &= \bar{F}_1(t) + \int_0^t \bar{F}_2(t-s) f_1(s) ds \\
 &= e^{-\lambda_1 t} + \int_0^t e^{-\lambda_2(t-s)} \lambda_1 e^{-\lambda_1 s} ds \\
 &= \frac{\lambda_1}{\lambda_1 - \lambda_2} e^{-\lambda_2 t} - \frac{\lambda_2}{\lambda_1 - \lambda_2} e^{-\lambda_1 t}
 \end{aligned}$$

The shorthand notation for the system's survival function should be obvious. The system has an $\text{EXP}(\lambda_1)$ life from the active component and an additional $\text{EXP}(\lambda_2)$ life from the spare. So the shorthand notation is

$$\text{EXP}(\lambda_1) + \text{EXP}(\lambda_2).$$

B. SUMS OF EXPONENTIALS WITH WEIGHT BETWEEN ZERO AND ONE

The examples given in the previous chapter only involved exponential lives with weight one. Now we will look at some structures, whose survival function has a shorthand notation which includes the MIX-notation and/or the ZERO-distribution.

1. Parallel System with Dissimilar Failure Rates

The notion of a parallel system has been introduced in Chapter III.A.2 . We now look at the case where

$$T_1 \sim \text{EXP}(\lambda_1) \text{ and } T_2 \sim \text{EXP}(\lambda_2).$$

Then

$$T = \max(T_1, T_2)$$

$$\bar{F}(t) = P(\max(T_1, T_2) > t)$$

$$= 1 - P(\max(T_1, T_2) \leq t)$$

$$= 1 - P(T_1 \leq t, T_2 \leq t)$$

Assuming independence of the two components

$$\bar{F}(t) = 1 - P(T_1 \leq t) P(T_2 \leq t)$$

$$= 1 - F_1(t) F_2(t)$$

$$= 1 - (1 - e^{-\lambda_1 t}) (1 - e^{-\lambda_2 t})$$

$$= 1 - (1 - e^{-\lambda_1 t} - e^{-\lambda_2 t} + e^{-(\lambda_1 + \lambda_2)t})$$

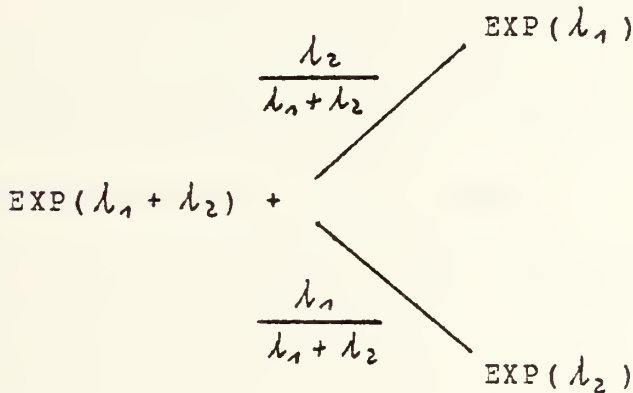
$$= e^{-\lambda_1 t} + e^{-\lambda_2 t} - e^{-(\lambda_1 + \lambda_2)t}.$$

To find the shorthand notation of the system consider all the ways which lead to the survival of the system:

- either both components survive
- or component 1 fails and component 2 survives
- or component 2 fails and component 1 survives.

If one component fails and one survives, in $\frac{\lambda_2}{\lambda_1 + \lambda_2}$ fraction of the cases the survivor will be component 1 and in $\frac{\lambda_1}{\lambda_1 + \lambda_2}$ fraction of the cases it will be component 2.

This can graphically be represented as



Making use of the MIX-notation the shorthand notation then is

$$\text{EXP}(\lambda_1 + \lambda_2) + \text{MIX}\left[\frac{\lambda_2}{\lambda_1 + \lambda_2} \text{EXP}(\lambda_1), \frac{\lambda_1}{\lambda_1 + \lambda_2} \text{EXP}(\lambda_2)\right]$$

and using the distributive property it becomes

$$\begin{aligned}
 &\text{MIX}\left[\frac{\lambda_2}{\lambda_1 + \lambda_2} (\text{EXP}(\lambda_1) + \text{EXP}(\lambda_1 + \lambda_2)), \right. \\
 &\quad \left. \frac{\lambda_1}{\lambda_1 + \lambda_2} (\text{EXP}(\lambda_2) + \text{EXP}(\lambda_1 + \lambda_2)) \right].
 \end{aligned}$$

As a check to see that this shorthand notation represents the survival function of the system, we derive the survival function from the shorthand notation:

$$\begin{aligned}\bar{F}(t) &= \frac{\lambda_2}{\lambda_1 + \lambda_2} \left(e^{-\lambda_1 t} + \int_0^t e^{-(\lambda_1 + \lambda_2)(t-s)} \lambda_1 e^{-\lambda_1 s} ds \right) \\ &\quad + \frac{\lambda_1}{\lambda_1 + \lambda_2} \left(e^{-\lambda_2 t} + \int_0^t e^{-(\lambda_1 + \lambda_2)(t-s)} \lambda_2 e^{-\lambda_2 s} ds \right) \\ &= e^{-\lambda_1 t} + e^{-\lambda_2 t} - e^{-(\lambda_1 + \lambda_2)t}.\end{aligned}$$

This verifies that the shorthand notation indeed represents the system's survival function.

2. Series System with One Spare

Let us now look at a two-component series system, whose components have dissimilar failure rates with one component having a spare:

Component 1 has the constant failure rate λ_1 and component 2 and the spare have the constant failure rate λ_2 . The spare can only replace component 2.

Let

$\bar{F}_1(t)$: the survival function of component 1

$\bar{F}_2(t)$: the survival function of the standby system component 2 with its spare.

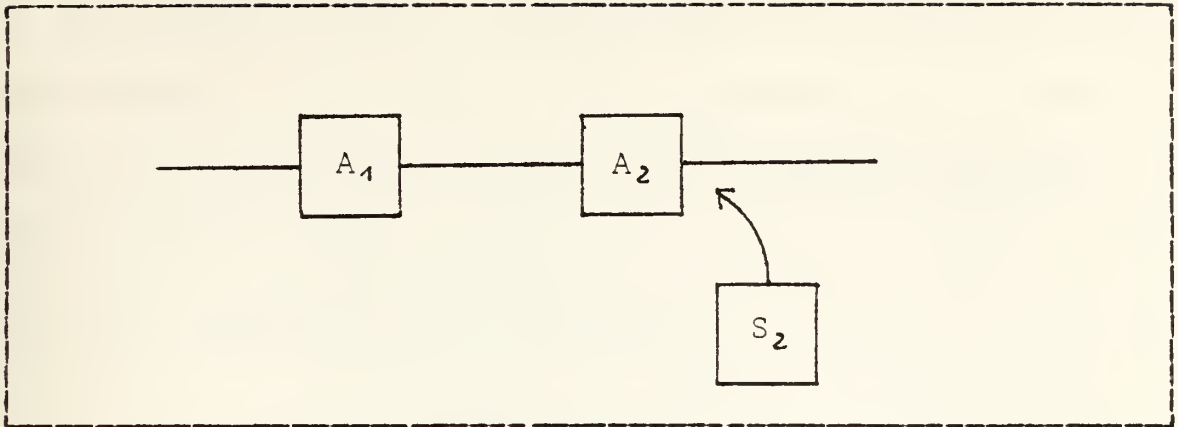


Figure 5: Series System with one Spare

The survival function for a standby system was derived in Chapter II.B. Therefore

$$\begin{aligned}
 \bar{F}_2(t) &= e^{-\lambda_2 t} + \int_0^t e^{-\lambda_2(t-s)} \lambda_2 e^{-\lambda_2 s} ds \\
 &= e^{-\lambda_2 t} + \lambda_2 e^{-\lambda_2 t} \int_0^t ds \\
 &= (1 + \lambda_2 t) e^{-\lambda_2 t} .
 \end{aligned}$$

Now $\bar{F}_1(t) = e^{-\lambda_1 t}$

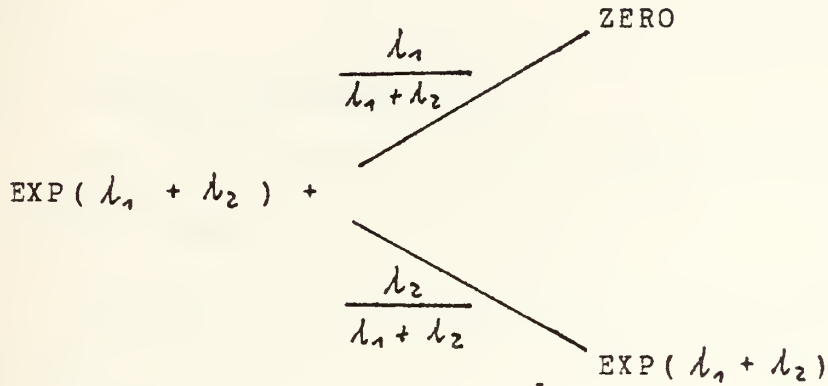
Then
$$\begin{aligned}
 \bar{F}(t) &= \bar{F}_1(t) \bar{F}_2(t) \\
 &= (1 + \lambda_2 t) e^{-(\lambda_1 + \lambda_2)t} .
 \end{aligned}$$

To translate the survival function into shorthand notation, let us consider the ways in which the system can survive:

- either both components survive
- or component 2 fails and its spare survives.

If one component fails, in $\frac{\lambda_1}{\lambda_1 + \lambda_2}$ fraction of the time it will be component 1, which means that the system will not survive; in $\frac{\lambda_2}{\lambda_1 + \lambda_2}$ fraction of the time the failing component will be component 2.

This can graphically be represented as



Using the MIX-notation the survival function then is

$$\begin{aligned}
 & \text{EXP}(\lambda_1 + \lambda_2) + \text{MIX}\left[\frac{\lambda_1}{\lambda_1 + \lambda_2} \text{ZERO}, \frac{\lambda_2}{\lambda_1 + \lambda_2} \text{EXP}(\lambda_1 + \lambda_2)\right] \\
 &= \text{MIX}\left[\frac{\lambda_1}{\lambda_1 + \lambda_2} (\text{ZERO} + \text{EXP}(\lambda_1 + \lambda_2)), \right. \\
 & \quad \left. \frac{\lambda_2}{\lambda_1 + \lambda_2} (\text{EXP}(\lambda_1 + \lambda_2) + \text{EXP}(\lambda_1 + \lambda_2)) \right] \\
 &= \text{MIX}\left[\frac{\lambda_1}{\lambda_1 + \lambda_2} (\text{EXP}(\lambda_1 + \lambda_2)), \right. \\
 & \quad \left. \frac{\lambda_2}{\lambda_1 + \lambda_2} (\text{EXP}(\lambda_1 + \lambda_2) + \text{EXP}(\lambda_1 + \lambda_2)) \right].
 \end{aligned}$$

To prove, that the shorthand notation does represent the survival function, we derive the latter from the shorthand:

$$\bar{F}(t) = \frac{\lambda_1}{\lambda_1 + \lambda_2} e^{-(\lambda_1 + \lambda_2)t} + \frac{\lambda_2}{\lambda_1 + \lambda_2} \left(e^{-(\lambda_1 + \lambda_2)t} + \right.$$

$$\int_0^t e^{-(\lambda_1 + \lambda_2)(t-s)} (\lambda_1 + \lambda_2) e^{-(\lambda_1 + \lambda_2)s} ds$$

$$= (1 + \lambda_2 t) e^{-(\lambda_1 + \lambda_2)t}.$$

This is the previously found result and this verifies, that the shorthand notation does represent the system's survival function.

3. Two-out-of-Three System

As a last example in this chapter, we will look at a Two-out-of- Three system.

Consider a three component system, whose components have constant failure rates λ_1 , λ_2 and λ_3 respectively. The system is functioning, as long as two out of three components are functioning (see Fig. 6).

In other words, the system is functioning as long as there is a path through the system .

Alternatively, the system can be visualized as a parallel-series system (compare Fig. 7).

The survival function of the system is

$$\begin{aligned} \bar{F}(t) &= P(T_1 > t \wedge T_2 > t) + P(T_1 > t \wedge T_3 > t) \\ &\quad + P(T_2 > t \wedge T_3 > t) \\ &\quad - P((T_1 > t \wedge T_2 > t) \wedge (T_1 > t \wedge T_3 > t)) \\ &\quad - P((T_1 > t \wedge T_2 > t) \wedge (T_2 > t \wedge T_3 > t)) \\ &\quad - P((T_1 > t \wedge T_3 > t) \wedge (T_2 > t \wedge T_3 > t)) \end{aligned}$$

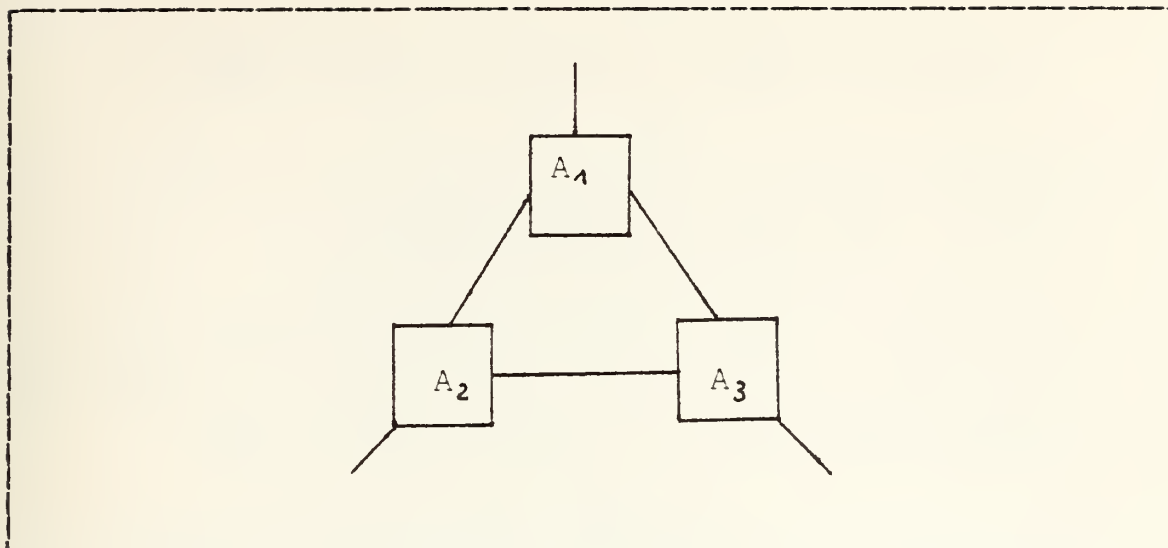


Figure 6: Two-out-of-Three System

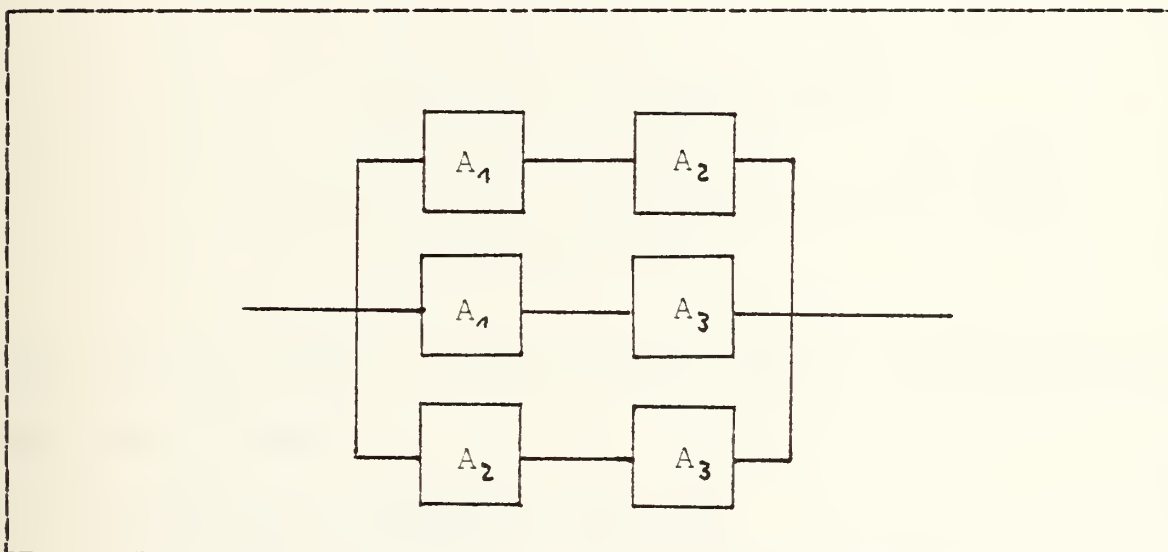


Figure 7: Two-out-of-Three System

$$\begin{aligned}
 &+ P((T_1 > t \wedge T_2 > t) \wedge (T_1 > t \wedge T_3 > t) \\
 &\quad \wedge P(T_2 > t \wedge T_3 > t)) .
 \end{aligned}$$

Thus

$$\begin{aligned}\bar{F}(t) &= P(T_1 > t \wedge T_2 > t) + P(T_1 > t \wedge T_3 > t) \\ &\quad + P(T_2 > t \wedge T_3 > t) \\ &\quad - 3P(T_1 > t \wedge T_2 > t \wedge T_3 > t) \\ &\quad + P(T_1 > t \wedge T_2 > t \wedge T_3 > t)\end{aligned}$$

Therefore, and assuming independence of the components,

$$\begin{aligned}\bar{F}(t) &= P(T_1 > t) P(T_2 > t) + P(T_1 > t) P(T_3 > t) \\ &\quad + P(T_2 > t) P(T_3 > t) \\ &\quad - 3P(T_1 > t) P(T_2 > t) P(T_3 > t) \\ &\quad + P(T_1 > t) P(T_2 > t) P(T_3 > t) \\ &= P(T_1 > t) P(T_2 > t) + P(T_1 > t) P(T_3 > t) \\ &\quad + P(T_2 > t) P(T_3 > t) \\ &\quad - 2P(T_1 > t) P(T_2 > t) P(T_3 > t) \\ &= e^{-(\lambda_1 + \lambda_2)t} + e^{-(\lambda_1 + \lambda_3)t} + e^{-(\lambda_2 + \lambda_3)t} \\ &\quad - 2e^{-(\lambda_1 + \lambda_2 + \lambda_3)t}.\end{aligned}$$

Now let us consider all the possible ways, in which the system can survive:

- either all components survive
- or component 1 fails and component 2 and 3 survive
- or component 2 fails and component 1 and 3 survive

- or component 3 fails and component 1 and 2

survive.

If a component fails and the other two survive, in $\frac{\lambda_i}{\lambda_1 + \lambda_2 + \lambda_3}$ fraction of the time it will be component i , $i = 1, 2, 3$.

This can graphically be represented as

$$\text{EXP}(\lambda_1 + \lambda_2 + \lambda_3) + \frac{\frac{\lambda_1}{\lambda_1 + \lambda_2 + \lambda_3} \text{EXP}(\lambda_2 + \lambda_3) + \frac{\lambda_2}{\lambda_1 + \lambda_2 + \lambda_3} \text{EXP}(\lambda_1 + \lambda_3) + \frac{\lambda_3}{\lambda_1 + \lambda_2 + \lambda_3} \text{EXP}(\lambda_1 + \lambda_2)}{\text{EXP}(\lambda_1 + \lambda_2 + \lambda_3)}$$

The shorthand notation then is

$$\begin{aligned} & \text{EXP}(\lambda_1 + \lambda_2 + \lambda_3) + \text{MIX} \left[\frac{\lambda_1}{\lambda_1 + \lambda_2 + \lambda_3} \text{EXP}(\lambda_2 + \lambda_3), \right. \\ & \quad \frac{\lambda_2}{\lambda_1 + \lambda_2 + \lambda_3} \text{EXP}(\lambda_1 + \lambda_3), \\ & \quad \left. \frac{\lambda_3}{\lambda_1 + \lambda_2 + \lambda_3} \text{EXP}(\lambda_1 + \lambda_2) \right], \\ & = \text{MIX} \left[\frac{\lambda_1}{\lambda_1 + \lambda_2 + \lambda_3} (\text{EXP}(\lambda_2 + \lambda_3) + \text{EXP}(\lambda_1 + \lambda_2 + \lambda_3)), \right. \\ & \quad \frac{\lambda_2}{\lambda_1 + \lambda_2 + \lambda_3} (\text{EXP}(\lambda_1 + \lambda_3) + \text{EXP}(\lambda_1 + \lambda_2 + \lambda_3)), \\ & \quad \left. \frac{\lambda_3}{\lambda_1 + \lambda_2 + \lambda_3} (\text{EXP}(\lambda_1 + \lambda_2) + \text{EXP}(\lambda_1 + \lambda_2 + \lambda_3)) \right]. \end{aligned}$$

Again, as a check that the shorthand notation represents the survival function, let us derive the survival function from the shorthand notation:

$$\begin{aligned}
 F(t) &= \frac{\lambda_1}{\lambda_1 + \lambda_2 + \lambda_3} \left[e^{-(\lambda_2 + \lambda_3)t} \right. \\
 &\quad \left. + \int_0^t e^{-(\lambda_1 + \lambda_2 + \lambda_3)(t-s)} (\lambda_2 + \lambda_3) e^{-(\lambda_2 + \lambda_3)s} ds \right] \\
 &+ \frac{\lambda_2}{\lambda_1 + \lambda_2 + \lambda_3} \left[e^{-(\lambda_1 + \lambda_3)t} \right. \\
 &\quad \left. + \int_0^t e^{-(\lambda_1 + \lambda_2 + \lambda_3)(t-s)} (\lambda_1 + \lambda_3) e^{-(\lambda_1 + \lambda_3)s} ds \right] \\
 &+ \frac{\lambda_3}{\lambda_1 + \lambda_2 + \lambda_3} \left[e^{-(\lambda_1 + \lambda_2)t} \right. \\
 &\quad \left. + \int_0^t e^{-(\lambda_1 + \lambda_2 + \lambda_3)(t-s)} (\lambda_1 + \lambda_2) e^{-(\lambda_1 + \lambda_2)s} ds \right] \\
 &= e^{-(\lambda_2 + \lambda_3)t} + e^{-(\lambda_1 + \lambda_3)t} + e^{-(\lambda_1 + \lambda_2)t} \\
 &\quad - 2 e^{-(\lambda_1 + \lambda_2 + \lambda_3)t}
 \end{aligned}$$

The result again proves that the shorthand notation indeed represents the survival function of the system.

IV. IMPLEMENTING THE SHORTHAND ON THE TI-59

The concept of a reliability shorthand is introduced in the course "Reliability and Weapons System Effectiveness Measurements", OA 4302, at the Naval Postgraduate School, Monterey. Most students taking the course are in the Operations Research (OR) - Curriculum.

The choice of the TI-59 as the computing device, on which the shorthand was to be implemented, was based on the fact, that each student in the OR-Curriculum is issued a TI-59 for use in basic probability and statistics courses. Thus almost every student at the Naval Postgraduate School, who is introduced to the shorthand, is familiar with the TI-59 and has access to such a calculator.

A program, that uses the shorthand notation, times to failure and failure rates as input, should

- calculate the survival probability of basic structures / small systems and
- require moderate computation time.

To achieve these requirements it was decided to incorporate all solutions for the convolution of up to four exponential random variables in the program. The formulas that were used are given in Appendix A.

Two programs are provided in this paper.

Program 1 can be used when all rates are dissimilar or all are the same. It uses the formulas on pages 37 and 38 only.

Program 2 can be used for the general case. It makes use of all the formulas given in Appendix A. The program includes a sorting routine that determines the applicable formula from the entered failure rates.

A user guide to the two programs is provided in Appendix B.

V. SUMMARY

There is a reliability shorthand that denotes the survival function of a system, assuming that the failure rates of all components are constant.

This shorthand can be implemented on the TI-59 handheld calculator. With failure rates, time to failure and shorthand as input the TI-59 calculates the survival probability of the system.

Knowledge of calculus is not necessary to use this method, whereas the standard procedure, finding the survival probability by convolution, requires knowledge of integral calculus.

The choice of the TI-59 as the computing device for the implementation of the shorthand, though, implied limitations; the number of failure rates is limited due to the limited storage capacity of the TI-59, and computing times are comparatively long. The TI-59 can therefore only be used for smaller systems, preferably for the solution of classroom problems.

For the solution of larger problems, the shorthand should be implemented on a state-of-the-art personal

computer using a general algorithm for the convolution of any number of exponential random variables.

APPENDIX A

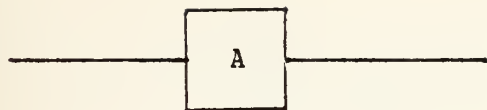
CONVOLUTION FORMULAS

Appendix A contains formulas for the convolution of up to four exponential random variables.

For the two special cases, when all random variables have the same failure rate and all have different failure rates, general formulas for the convolution of any number of exponential random variables are given.

These formulas are used in the two TI-59 programs provided in Appendix B.

System:



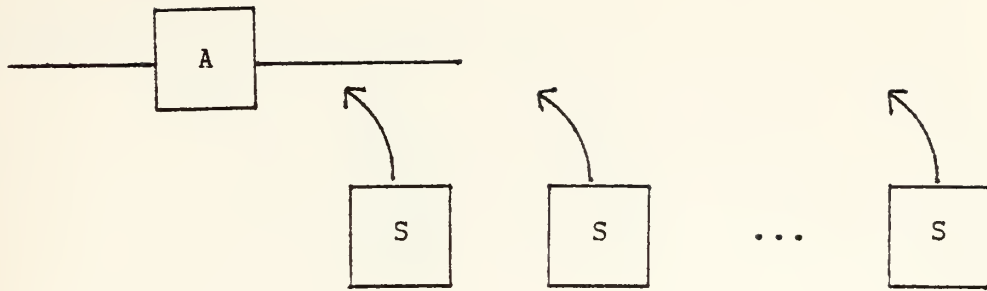
Shorthand: $\text{EXP}(\lambda)$

Survival Function: $\bar{F}(t) = e^{-\lambda t}$

Description:

A single active component with constant failure rate λ .

System:



Shorthand: $\text{EXP}(\lambda) + \text{EXP}(\lambda) + \dots + \text{EXP}(\lambda)$

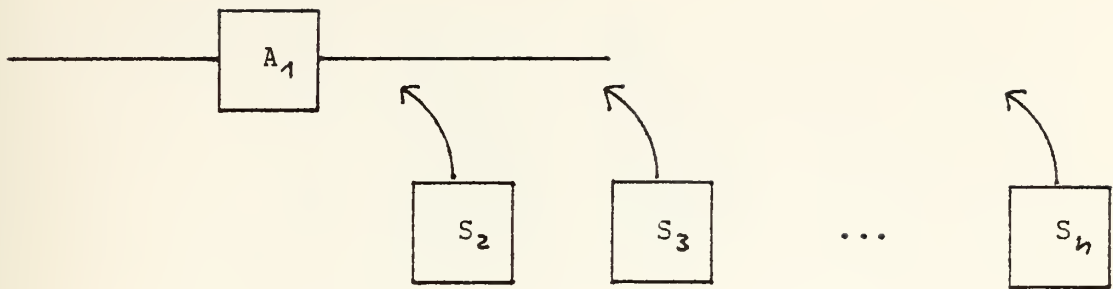
$$\text{Survival Function: } \bar{F}(t) = \left(\frac{(\lambda t)^0}{0!} + \frac{(\lambda t)^1}{1!} + \dots + \frac{(\lambda t)^{n-1}}{(n-1)!} \right) e^{-\lambda t}$$

$$= \sum_{i=1}^n \frac{(\lambda t)^{i-1}}{(i-1)!} e^{-\lambda t}$$

Description:

A single active component with constant failure rate is supported by $n-1$ identical spares.

System:



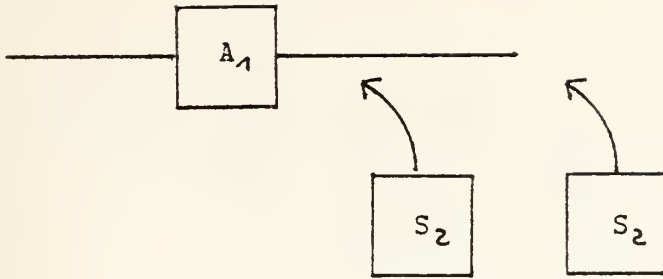
Shorthand: $\text{EXP}(\lambda_1) + \text{EXP}(\lambda_2) + \dots + \text{EXP}(\lambda_n)$

Survival Function:
$$\bar{F}(t) = \sum_{i=1}^n \left(\prod_{j \neq i} \frac{\lambda_j}{\lambda_j - \lambda_i} e^{-\lambda_i t} \right)$$

Description:

A single active component with constant failure rate is supported by $n-1$ spares. The active component and the spares have all constant, but dissimilar failure rates.

System:



Shorthand: $\text{EXP}(\lambda_1) + \text{EXP}(\lambda_2) + \text{EXP}(\lambda_2)$

Survival Function: $\bar{F}(t) = A e^{-\lambda_1 t} + (B + Ct) e^{-\lambda_2 t}$

$$\text{where } A = \frac{\lambda_2^2}{(\lambda_2 - \lambda_1)^2}$$

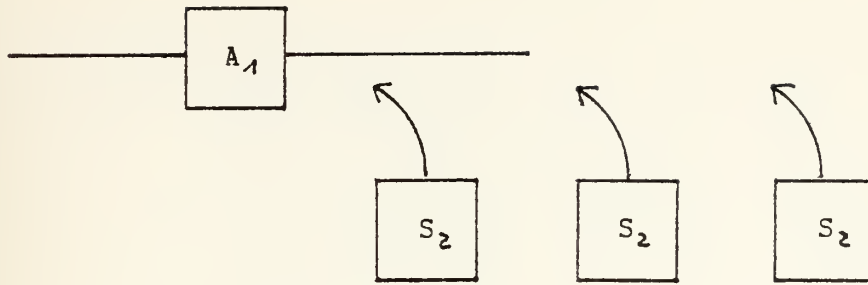
$$B = 1 - A$$

$$C = \frac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2}$$

Description:

A single active component with constant failure rate λ_1 is supported by two spares with identical constant failure rate λ_2 .

System:



Shorthand: $\text{EXP}(\lambda_1) + \text{EXP}(\lambda_2) + \text{EXP}(\lambda_2) + \text{EXP}(\lambda_2)$

Survival Function: $\bar{F}(t) = A e^{-\lambda_1 t} + (B + Ct + Dt^2) e^{-\lambda_2 t}$

$$\text{where } A = \frac{\lambda_2^3}{(\lambda_2 - \lambda_1)^3}$$

$$B = 1 - A$$

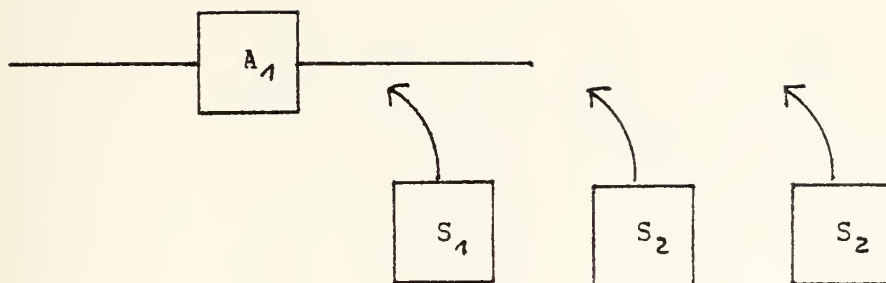
$$C = \lambda_2 - \frac{\lambda_2^3}{(\lambda_1 - \lambda_2)^2}$$

$$D = \frac{\lambda_1 \lambda_2^2}{2 (\lambda_1 - \lambda_2)}$$

Description:

A single active component with constant failure rate λ_1 is supported by three spares with identical constant failure rate λ_2 .

System:



Shorthand: $\text{EXP}(\lambda_1) + \text{EXP}(\lambda_1) + \text{EXP}(\lambda_2) + \text{EXP}(\lambda_2)$

Survival Function: $F(t) = (A + Bt)e^{-\lambda_1 t} + (C + Dt)e^{-\lambda_2 t}$

$$\text{where } A = \frac{\lambda_2^3 - 3\lambda_2^2 \lambda_1}{(\lambda_2 - \lambda_1)^3}$$

$$B = \frac{\lambda_1 \lambda_2^2}{(\lambda_2 - \lambda_1)^2}$$

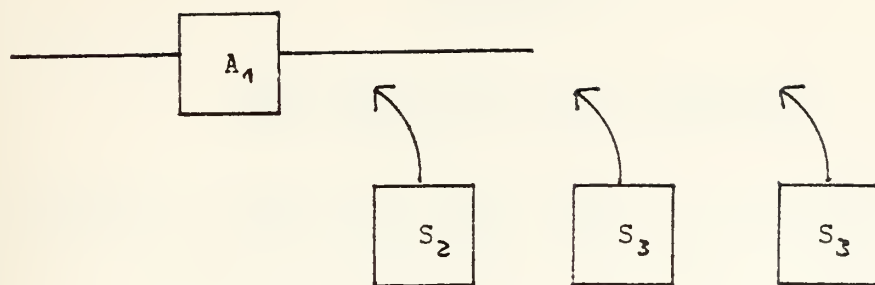
$$C = 1 - A$$

$$D = \frac{\lambda_1^2 \lambda_2}{(\lambda_1 - \lambda_2)^2}$$

Description:

A single active component with constant failure rate λ_1 is supported by one identical spare and two spares with dissimilar, constant failure rate λ_2 .

System:



Shorthand: $\text{EXP}(\lambda_1) + \text{EXP}(\lambda_2) + \text{EXP}(\lambda_3) + \text{EXP}(\lambda_3)$

Survival Function: $\bar{F}(t) = A e^{-\lambda_1 t} + B e^{-\lambda_2 t} + (C + Dt) e^{-\lambda_3 t}$

$$\text{where } A = \frac{\lambda_2 \lambda_3^2}{(\lambda_2 - \lambda_1)(\lambda_3 - \lambda_1)^2}$$

$$B = \frac{\lambda_1 \lambda_3^2}{(\lambda_1 - \lambda_2)(\lambda_3 - \lambda_2)^2}$$

$$C = \frac{\lambda_1 \lambda_2}{(\lambda_1 - \lambda_3)(\lambda_2 - \lambda_3)} + \frac{\lambda_1 \lambda_2 \lambda_3}{(\lambda_1 - \lambda_2)} \left(\frac{1}{(\lambda_1 - \lambda_3)^2} - \frac{1}{(\lambda_2 - \lambda_3)^2} \right)$$

$$D = \frac{\lambda_1 \lambda_2 \lambda_3}{(\lambda_1 - \lambda_3)(\lambda_2 - \lambda_3)}$$

Description:

A single active component with constant failure rate λ_1 has three spares. One spare has constant failure rate λ_2 , two spares are identical with constant failure rate λ_3 .

APPENDIX B

USER GUIDE TO TI-59 PROGRAMS

Appendix B contains a user guide to two TI-59 programs, which use reliability shorthand and failure rates as input to compute the survival probability of a system.

PROGRAM 1 is designed for the two special cases where the reliability shorthand is of the form

$$\text{EXP}(\lambda) + \text{EXP}(\lambda) + \dots + \text{EXP}(\lambda)$$

or

$$\text{EXP}(\lambda_1) + \text{EXP}(\lambda_2) + \dots + \text{EXP}(\lambda_n).$$

In the first case the number of terms is not limited, whereas in the second case the number of terms is limited to 40 due to limited storage capacity of the TI-59. In this case the number of terms can be increased to 70 by entering 9 in the display and pressing 2nd Op 1 7 .

PROGRAM 2 is designed to solve problems of the kind, that were introduced in Chapter III.B. . Due to limited memory of the TI-59 the number of exponential terms under one weight in shorthand notation is limited to four.

All results will be printed, if the TI-59 is connected to a TI PC-100A or TI PC-100C printer.

PROGRAM 1 : Procedure

1. Use any library module. Read in program 1 (side 1 of the magnetic card)
2. Press 2nd C' to initialize.
3. Enter n , the number of exponential terms to be convolved, in the display and press A.
4. Enter time t and press B.
5. Enter λ_i and press C . When all failure rates are the same, enter λ only once.
6. a) To find the survival probability of the system, when all failure rates are the same, press 2nd A'.
b) To find the survival probability of the system, when all failure rates are dissimilar, press 2nd B'.

PROGRAM 1 : Sample Problems

1. Find the survival probability of a parallel system

(compare Chapter III.A.2)

a) $\lambda = .3$, $t = 7$, $n = 2$

b) Shorthand notation:

$$\text{EXP}(.6) + \text{EXP}(.3)$$

c)	Enter	Comment	Press	Display
		Initialize	C'	0
	2	n	A	0
	7	t	B	7
	.6	2 λ	C	.3
	.3	λ	C	.3
		$\bar{F}(t)$	B'	.2299172797

calculation takes 13 seconds

2. Find the survival probability of a standby-system with dissimilar components (compare Chapter III.A.3) .

a) $\lambda_1 = .4$, $\lambda_2 = .5$, $t = 6$, $n = 2$

b) Shorthand notation:

EXP(.4) + EXP(.5)

c)	Enter	Comment	Press	Display
		Initialize	C'	0
	2	n	A	0
	6	t	B	6
	.4	λ_1	C	.4
	.5	λ_2	C	.5
		$\bar{F}(t)$	B'	.254441493

calculation takes 13 seconds

3. Find the survival probability of a standby-system with one active component and four similar spares.

a) $\lambda = .3$, $t = 7$, $n = 5$

b) Shorthand notation:

$\text{EXP}(.3) + \text{EXP}(.3) + \text{EXP}(.3) + \text{EXP}(.3) + \text{EXP}(.3)$

c)	Enter	Comment	Press	Display
		Initialize	C'	0
	5	n	A	0
	7	t	B	7
	.3	λ	C	.3
	.	$\bar{F}(t)$	A'	.9378738848

calculation takes 9 seconds

PROGRAM 2 : Procedure

CASE I : To find the convolution of up to four exponential random variables.

1. Use any library module.

Re-Partition (enter 2 in the display, press 2nd Op 17).

Read in all four sides of the magnetic card.

2. Press 2nd C' to initialize.
3. Enter n , the number of exponential terms to be convolved, in the display and press A.
4. Enter time t and press B.
5. Enter λ_i and press C (n entries) .

REMARK: Failure rates, which appear only once in the expression, have to be entered before failure rates, that appear several times.

6. To find the survival probability of the system press E.

PROGRAM 2, CASE I : Sample Problems

(1) Shorthand notation

$$\text{EXP}(\lambda_1) + \text{EXP}(\lambda_2) + \text{EXP}(\lambda_2)$$

Sample values : $\lambda_1 = .3$, $\lambda_2 = .4$, $t = 7$

Procedure :

Enter	Comment	Press	Display
	Initialize	C'	0
3	n	A	0
7	t	B	7
.3	λ_1	C	.3
.4	λ_2	C	.4
.4	λ_2	C	.4
	$\bar{F}(t)$	E	.5363473866

calculation takes 14 seconds

(2) Shorthand notation

$$\text{EXP}(\lambda_1) + \text{EXP}(\lambda_2) + \text{EXP}(\lambda_2) + \text{EXP}(\lambda_2)$$

Sample values : $\lambda_1 = .2$, $\lambda_2 = .4$, $t = 3$

Procedure :

Enter	Comment	Press	Display
	Initialize	C'	0
4	n	A	0
3	t	B	3
.2	λ_1	C	.2
.4	λ_2	C	.4
.4	λ_2	C	.4
.4	λ_2	C	.4
	$\bar{F}(t)$	E	.9809746099

calculation takes 20 seconds

(3) Shorthand notation

$$\text{EXP}(\lambda_1) + \text{EXP}(\lambda_1) + \text{EXP}(\lambda_2) + \text{EXP}(\lambda_2)$$

Sample values : $\lambda_1 = .4$, $\lambda_2 = .3$, $t = 5$

Procedure :

Enter	Comment	Press	Display
	Initialize	C'	0
4	n	A	0
5	t	B	5
.4	λ_1	C	.4
.4	λ_1	C	.4
.3	λ_2	C	.3
.3	λ_2	C	.3
	$\bar{F}(t)$	E	.9029040721

calculation takes 20 seconds

(4) Shorthand notation

$$\text{EXP}(\lambda_1) + \text{EXP}(\lambda_2) + \text{EXP}(\lambda_3) + \text{EXP}(\lambda_3)$$

Sample values : $\lambda_1 = .1$, $\lambda_2 = .3$, $\lambda_3 = .5$,

$$t = 10$$

Procedure :

Enter	Comment	Press	Display
	Initialize	C'	0
4	n	A	0
10	t	B	10
.1	λ_1	C	.1
.3	λ_2	C	.3
.5	λ_3	C	.5
.5	λ_3	C	.5
	$\bar{F}(t)$	E	.7312684703

calculation takes 25 seconds

PROGRAM 2 : Procedure

CASE II : to solve problems of the kind, that were
introduced in Chapter III.B. .

1. Derive the system's shorthand notation. Find either the
 - graphical representation or
 - the MIX-notation .

2. Use any library module.

Re-Partition (enter 2 in the display, press 2nd Op 17).

Read in all four sides of the magnetic card.

3. Press 2nd C' to initialize.
4. Enter time t and press B.
5. Repeat the following steps for each path of the graphical representation, i.e. for each convolution in the MIX-notation.

- a) Enter n , the number of exponential terms to be convolved, in the display and press A.

- b) Enter λ_i and press C.

REMARK: Failure rates, which appear only once in the expression, have to be entered before failure rates, that appear several times.

- c) Enter p_i , the weight in the i th path, and press D.

- d) To find the part of the system's survival probability, that is contributed by the i th path, press E.

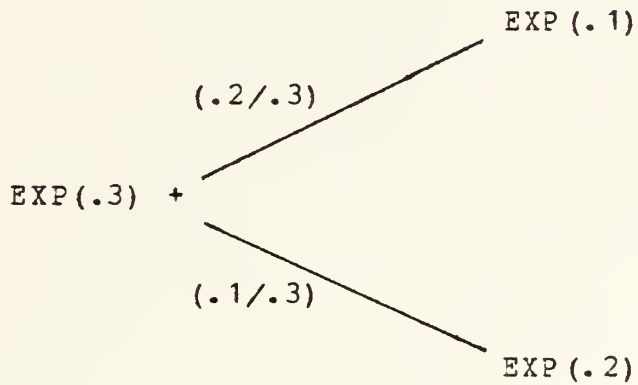
6. To find the survival probability of the system
press 2nd E'.

PROGRAM 2, CASE II : Sample Problems

1. Find the survival probability of a parallel system
with dissimilar failure rates (compare Chapter III.B.1).

a) $\lambda_1 = .1$, $\lambda_2 = .2$, $t = 2$

b) Shorthand notation



$$\bar{F}(t) = \text{MIX} [\quad (.2/.3) (\text{EXP}(.1) + \text{EXP}(.3) , \\ \quad (.1/.3) (\text{EXP}(.2) + \text{EXP}(.3))].$$

c)

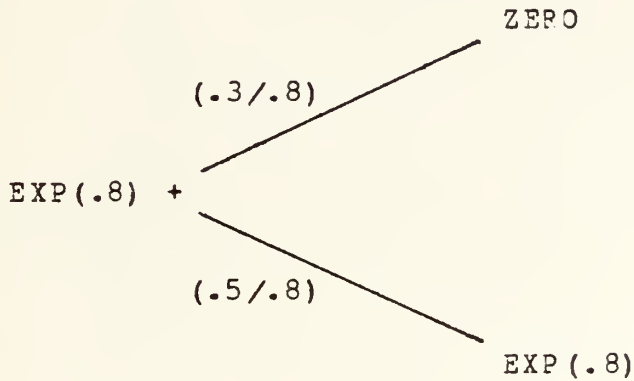
Procedure :

Enter	Comment	Press	Display
	Initialize	C'	0
2	t	B	2
2	n_1	A	0
.1	h_1	C	.1
.3	$h_1 + h_2$	C	.3
(.2/.3)	P_1	D	.6666666667
		E	.635793541
2	n_2	A	0
.2	h_2	C	.2
.3	$h_1 + h_2$	C	.3
(.1/.3)	P_2	D	.3333333333
		E	.304445622
	$\bar{F}(t)$	E'	.940239163

2. Find the survival probability of a series system with one spare as introduced in Chapter III.B.2 .

a) $\lambda_1 = .3$, $\lambda_2 = .5$, $t = 7$

b) Shorthand notation



$$\bar{F}(t) = \text{MIX}[(.3/.8)(\text{EXP}(.8),$$

$$(.5/.8)(\text{EXP}(.8) + \text{EXP}(.8))].$$

c)

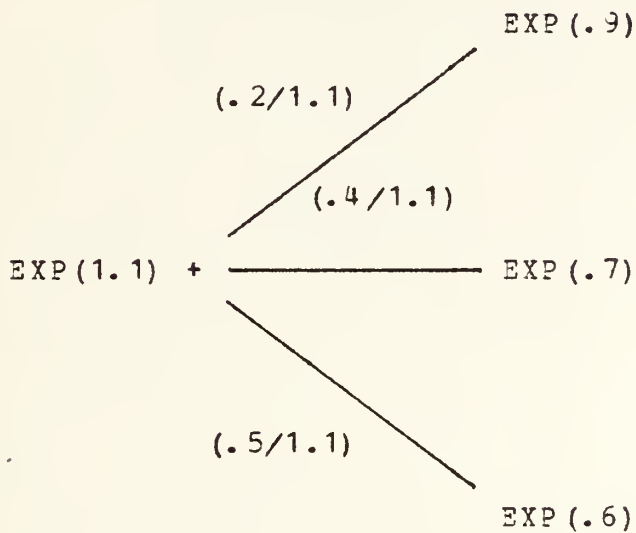
Procedure :

Enter	Comment	Press	Display
	Initialize	C'	0
7	t	B	7
1	n_1	A	0
.8	$\lambda_1 + \lambda_2$	C	.8
(.3/.8)	p_1	D	.375
		E	.0013866989
2	n_2	A	0
.8	$\lambda_1 + \lambda_2$	C	.8
.8	$\lambda_1 + \lambda_2$	C	.8
(.5/.8)	p_2	D	.625
		E	.0152536878
	$\overline{F}(t)$	E'	.0166403867

3. Find the survival probability of a Two-out-of-Three System as introduced in Chapter III.B.3 .

a) $\lambda_1 = .2$, $\lambda_2 = .4$, $\lambda_3 = .5$, $t = 9$

b) Shorthand notation



$$\begin{aligned} \bar{F}(t) = \text{MIX}[& (.2/1.1) (\text{EXP}(.9) + \text{EXP}(1.1)) , \\ & (.4/1.1) (\text{EXP}(.7) + \text{EXP}(1.1)) , \\ & (.5/1.1) (\text{EXP}(.6) + \text{EXP}(1.1))]. \end{aligned}$$

c)

Procedure :

Enter	Comment	Press	Display
	Initialize	C'	0
9	t	B	9
2	n_1	A	0
1.1	$\lambda_1 + \lambda_2 + \lambda_3$	C	1.1
.9	$\lambda_2 + \lambda_3$	C	.9
(.2/1.1)	P_1	D	.1818181818
		E	.0002624871
2	n_2	A	0
1.1	$\lambda_1 + \lambda_2 + \lambda_3$	C	1.1
.7	$\lambda_1 + \lambda_3$	C	.7
(.4/1.1)	P_2	D	.3636363636
		E	.0018043754
2	n_3	A	0
1.1	$\lambda_1 + \lambda_2 + \lambda_3$	C	1.1
.6	$\lambda_1 + \lambda_2$	C	.6
(.5/1.1)	P_3	D	.4545454545
		E	.0044892129
	$\bar{F}(t)$	E'	.0065560755

COMPUTER LISTINGS

PROGRAM 1

000	51	R/S	040	76	LBL	080	43	RCL
001	76	LBL	041	16	H*	081	57	05
002	18	C*	042	43	RCL	082	49	FRD
003	29	CP	043	20	20	083	04	04
004	25	CLR	044	65	X	084	61	GTO
005	47	CMS	045	43	RCL	085	28	LOG
006	91	RST	046	01	01	086	76	LBL
007	76	LBL	047	95	=	087	29	CP
008	11	A	048	42	STD	088	43	RCL
009	42	STD	049	02	02	089	02	02
010	00	00	050	00	0	090	94	+/-
011	75	-	051	01	1	091	22	INV
012	01	1	052	42	STD	092	23	LNK
013	95	=	053	04	04	093	65	X
014	42	STD	054	42	STD	094	71	SBR
015	09	09	055	05	05	095	47	CMS
016	29	CP	056	42	STD	096	95	=
017	02	2	057	18	18	097	99	PRT
018	00	0	058	76	LBL	098	51	R/S
019	42	STD	059	28	LOG	099	76	LBL
020	08	08	060	29	CP	100	17	B*
021	01	1	061	43	RCL	101	02	2
022	42	STD	062	09	09	102	00	0
023	17	17	063	67	EQ	103	42	STD
024	00	0	064	29	CP	104	08	08
025	42	STD	065	43	RCL	105	76	LBL
026	18	18	066	02	02	106	39	CDS
027	91	R/S	067	45	YK	107	02	2
028	76	LBL	068	43	RCL	108	00	0
029	12	B	069	05	05	109	42	STD
030	42	STD	070	55	+	110	06	06
031	01	01	071	43	RCL	111	01	1
032	91	R/S	072	04	04	112	42	STD
033	76	LBL	073	95	=	113	18	18
034	13	C	074	44	SUM	114	76	LBL
035	72	ST*	075	18	18	115	38	SIN
036	08	08	076	69	DP	116	29	CP
037	69	DP	077	39	39	117	73	RC*
038	28	28	078	69	DP	118	06	06
039	91	R/S	079	25	25	119	67	EQ

PROGRAM 1 continued

LABEL ADRESSES

120	36	PGM	160	36	PGM	002	18	C*
121	73	RC*	161	73	RC*	008	11	A
122	08	08	162	08	08	029	12	B
123	32	XIT	163	94	+/-	034	13	C
124	73	RC*	164	65	X	041	16	A*
125	06	06	165	43	RCL	059	28	LDG
126	22	INV	166	01	01	087	29	CP
127	67	EQ	167	95	=	100	17	B*
128	30	TAN	168	22	INV	106	39	CDS
129	69	DP	169	23	LNK	115	38	SIN
130	26	26	170	65	X	134	30	TAN
131	61	GTD	171	43	RCL	160	36	PGM
132	38	SIN	172	16	16	192	37	P/R
133	76	LBL	173	95	=	198	47	CMS
134	30	TAN	174	44	SUM			
135	49	PRD	175	18	18			
136	16	16	176	43	RCL			
137	75	-	177	00	00			
138	73	RC*	178	32	XIT			
139	08	08	179	43	RCL			
140	95	=	180	08	08			
141	35	1/X	181	75	-			
142	49	PRD	182	01	1			
143	16	16	183	09	9			
144	43	RCL	184	95	=			
145	00	00	185	67	EQ			
146	32	XIT	186	37	P/R			
147	43	RCL	187	69	DP			
148	06	06	188	28	28			
149	75	-	189	61	GTD			
150	01	1	190	39	CDS			
151	09	9	191	76	LBL			
152	95	=	192	37	P/R			
153	77	GE	193	71	SBR			
154	36	PGM	194	47	CMS			
155	69	DP	195	99	PRT			
156	26	26	196	91	R/S			
157	61	GTD	197	76	LBL			
158	38	SIN	198	47	CMS			
159	76	LBL	199	43	RCL			
			200	18	18			
			201	92	RTN			

PROGRAM 2

000	86	STF	040	69	DP	080	04	04
001	00	00	041	28	28	081	95	=
002	61	GTD	042	91	R/S	082	44	SUM
003	15	E	043	76	LBL	083	18	18
004	76	LBL	044	14	D	084	69	DP
005	18	C'	045	49	FRD	085	39	39
006	29	CP	046	17	17	086	69	DP
007	25	CLR	047	91	R/S	087	25	25
008	47	CMS	048	76	LBL	088	43	RCL
009	91	R/S	049	16	A'	089	05	05
010	76	LBL	050	43	RCL	090	49	FRD
011	11	A	051	10	10	091	04	04
012	42	STD	052	65	X	092	61	GTD
013	00	00	053	43	RCL	093	28	LDG
014	75	-	054	01	01	094	76	LBL
015	01	1	055	95	=	095	29	CP
016	95	=	056	42	STD	096	43	RCL
017	42	STD	057	02	02	097	02	02
018	09	09	058	00	0	098	94	+/-
019	29	CP	059	01	1	099	22	INV
020	01	1	060	42	STD	100	23	LNK
021	00	0	061	04	04	101	65	X
022	42	STD	062	42	STD	102	71	SBR
023	08	08	063	05	05	103	47	CMS
024	01	1	064	42	STD	104	95	=
025	42	STD	065	18	18	105	91	R/S
026	17	17	066	76	LBL	106	76	LBL
027	00	0	067	28	LDG	107	17	B'
028	42	STD	068	29	CP	108	01	1
029	18	18	069	43	RCL	109	00	0
030	91	R/S	070	09	09	110	42	STD
031	76	LBL	071	67	EQ	111	08	08
032	12	B	072	29	CP	112	76	LBL
033	42	STD	073	43	RCL	113	39	CDS
034	01	01	074	02	02	114	01	1
035	91	R/S	075	45	YX	115	00	0
036	76	LBL	076	43	RCL	116	42	STD
037	13	C	077	05	05	117	06	06
038	72	ST*	078	55	+	118	01	1
039	08	08	079	43	RCL	119	42	STD

PROGRAM 2 continued

120	16	16	160	36	PGM	200	91	R/S
121	76	LBL	161	69	DP	201	76	LBL
122	38	SIN	162	26	26	202	47	CMS
123	29	CP	163	61	GTD	203	43	RCL
124	73	RC*	164	38	SIN	204	18	18
125	06	06	165	76	LBL	205	65	X
126	67	EQ	166	36	PGM	206	43	RCL
127	36	PGM	167	73	RC*	207	17	17
128	73	RC*	168	08	08	208	95	=
129	08	08	169	94	+/-	209	99	PRT
130	32	X↑T	170	65	X	210	44	SUM
131	73	RC*	171	43	RCL	211	19	19
132	06	06	172	01	01	212	92	RTN
133	22	INV	173	95	=	213	76	LBL
134	67	EQ	174	22	INV	214	22	INV
135	30	TAN	175	23	LNX	215	43	RCL
136	69	DP	176	65	X	216	11	11
137	26	26	177	43	RCL	217	33	X²
138	61	GTD	178	16	16	218	55	÷
139	38	SIN	179	95	=	219	53	(
140	76	LBL	180	44	SUM	220	43	RCL
141	30	TAN	181	18	18	221	11	11
142	49	PRD	182	43	RCL	222	75	-
143	16	16	183	00	00	223	43	RCL
144	75	-	184	32	X↑T	224	10	10
145	73	RC*	185	43	RCL	225	54)
146	08	08	186	08	08	226	33	X²
147	95	=	187	75	-	227	95	=
148	35	1/X	188	09	9	228	42	STD
149	49	PRD	189	95	=	229	07	07
150	16	16	190	67	EQ	230	65	X
151	43	RCL	191	37	P/R	231	53	(
152	00	00	192	69	DP	232	43	RCL
153	32	X↑T	193	28	28	233	10	10
154	43	RCL	194	61	GTD	234	94	+/-
155	06	06	195	09	CDS	235	65	X
156	75	-	196	76	LBL	236	43	RCL
157	09	9	197	37	P/R	237	01	01
158	95	=	198	71	SBR	238	54)
159	77	GE	199	47	CMS	239	22	INV

PROGRAM 2 continued

240	23	LNK	280	18	18	320	01	1
241	95	=	281	71	SBR	321	85	+
242	42	STD	282	47	CMS	322	53	(
243	18	18	283	91	R/S	323	43	RCL
244	43	RCL	284	76	LBL	324	11	11
245	07	07	285	23	LNK	325	75	-
246	94	+/-	286	40	RCL	326	43	RCL
247	85	+	287	11	11	327	11	11
248	01	1	288	45	YX	328	45	YX
249	85	+	289	03	3	329	03	3
250	43	RCL	290	55	+	330	55	+
251	10	10	291	53	(331	53	(
252	65	X	292	43	RCL	332	43	RCL
253	43	RCL	293	11	11	333	10	10
254	11	11	294	75	-	334	75	-
255	55	+	295	43	RCL	335	43	RCL
256	53	(296	10	10	336	11	11
257	43	RCL	297	54)	337	54)
258	10	10	298	45	YX	338	33	X ²
259	75	-	299	03	3	339	54)
260	43	RCL	300	95	=	340	65	X
261	11	11	301	42	STD	341	43	RCL
262	54)	302	07	07	342	01	01
263	65	X	303	65	X	343	85	+
264	43	RCL	304	53	(344	43	RCL
265	01	01	305	43	RCL	345	10	10
266	95	=	306	10	10	346	65	X
267	65	X	307	94	+/-	347	43	RCL
268	53	(308	65	X	348	11	11
269	43	RCL	309	43	RCL	349	33	X ²
270	11	11	310	01	01	350	55	+
271	94	+/-	311	54)	351	53	(
272	65	X	312	22	INV	352	02	2
273	43	RCL	313	23	LNK	353	65	X
274	01	01	314	85	+	354	53	(
275	54)	315	53	(355	43	RCL
276	22	INV	316	43	RCL	356	10	10
277	23	LNK	317	07	07	357	75	-
278	95	=	318	94	+/-	358	43	RCL
279	44	SUM	319	85	+	359	11	11

PROGRAM 2 continued

360	54)	400	55	+	440	65	x
361	54)	401	53	(441	43	RCL
362	65	x	402	53	(442	01	01
363	43	RCL	403	43	RCL	443	54)
364	01	01	404	12	12	444	22	INV
365	33	X²	405	75	-	445	23	LNx
366	54)	406	43	RCL	446	95	=
367	65	x	407	10	10	447	42	STO
368	53	(408	54)	448	18	18
369	43	RCL	409	65	x	449	53	(
370	11	11	410	33	X²	450	43	RCL
371	94	+/-	411	54)	451	07	07
372	65	x	412	95	=	452	94	+/-
373	43	RCL	413	42	STO	453	85	+
374	01	01	414	07	07	454	01	1
375	54)	415	85	+	455	85	+
376	22	INV	416	43	RCL	456	43	RCL
377	23	LNx	417	10	10	457	10	10
378	95	=	418	65	x	458	33	X²
379	44	SUM	419	43	RCL	459	65	x
380	18	18	420	12	12	460	43	RCL
381	71	SBR	421	33	X²	461	12	12
382	47	CMS	422	55	+	462	55	+
383	91	R/S	423	53	(463	53	(
384	76	LBL	424	43	RCL	464	43	RCL
385	24	CE	425	12	12	465	10	10
386	43	RCL	426	75	-	466	75	-
387	12	12	427	43	RCL	467	43	RCL
388	65	x	428	10	10	468	12	12
389	33	X²	429	54)	469	54)
390	75	-	430	33	X²	470	33	X²
391	03	3	431	65	x	471	65	x
392	65	x	432	43	RCL	472	43	RCL
393	43	RCL	433	01	01	473	01	01
394	12	12	434	95	=	474	54)
395	33	X²	435	65	x	475	65	x
396	65	x	436	53	(476	53	(
397	43	RCL	437	43	RCL	477	43	RCL
398	10	10	438	10	10	478	12	12
399	95	=	439	94	+/-	479	94	+/-

PROGRAM 2 continued

480	65	X	520	53	(560	11	11
481	43	RCL	521	43	RCL	561	94	+/-
482	01	01	522	10	10	562	65	X
483	54)	523	94	+/-	563	43	RCL
484	22	INV	524	65	X	564	01	01
485	23	LNK	525	43	RCL	565	54)
486	95	=	526	01	01	566	22	INV
487	44	SUM	527	54)	567	23	LNK
488	18	18	528	22	INV	568	95	=
489	71	SBR	529	23	LNK	569	44	SUM
490	47	CMS	530	95	=	570	18	18
491	91	R/S	531	42	STD	571	43	RCL
492	76	LBL	532	18	18	572	10	10
493	25	CLR	533	43	RCL	573	65	X
494	43	RCL	534	10	10	574	43	RCL
495	11	11	535	65	X	575	11	11
496	65	X	536	43	RCL	576	55	+
497	43	RCL	537	12	12	577	53	(
498	12	12	538	33	X²	578	43	RCL
499	33	X²	539	55	+	579	10	10
500	95	=	540	53	(580	75	-
501	55	+	541	43	RCL	581	43	RCL
502	53	(542	10	10	582	12	12
503	43	RCL	543	75	-	583	54)
504	11	11	544	43	RCL	584	55	+
505	75	-	545	11	11	585	53	(
506	43	RCL	546	54)	586	43	RCL
507	10	10	547	55	+	587	11	11
508	54)	548	53	(588	75	-
509	55	+	549	43	RCL	589	43	RCL
510	53	(550	12	12	590	12	12
511	43	RCL	551	75	-	591	54)
512	12	12	552	43	RCL	592	95	=
513	75	-	553	11	11	593	42	STD
514	43	RCL	554	54)	594	07	07
515	10	10	555	33	X²	595	65	+
516	54)	556	95	=	596	43	RCL
517	33	X²	557	65	X	597	10	10
518	95	=	558	53	(598	65	X
519	65	X	559	43	RCL	599	43	RCL

PROGRAM 2 continued

600	11	11	640	12	12	680	86	STF
601	65	X	641	65	X	681	40	IND
602	43	RCL	642	43	RCL	682	00	00
603	12	12	643	01	01	683	00	0
604	55	÷	644	95	=	684	42	STD
605	53	(645	65	X	685	07	07
606	43	RCL	646	53	(686	43	RCL
607	10	10	647	43	RCL	687	11	11
608	75	-	648	12	12	688	32	X&T
609	43	RCL	649	94	+/-	689	87	IFF
610	11	11	650	65	X	690	01	01
611	54)	651	43	RCL	691	16	A*
612	65	X	652	01	01	692	87	IFF
613	53	(653	54)	693	02	02
614	53	(654	22	INV	694	48	EXC
615	43	RCL	655	23	LNK	695	87	IFF
616	10	10	656	95	=	696	03	03
617	75	-	657	44	SUM	697	49	PRD
618	43	RCL	658	18	18	698	87	IFF
619	12	12	659	71	SBR	699	04	04
620	54)	660	47	CMS	700	50	IXI
621	33	X²	661	91	R/S	701	76	LBL
622	35	1/X	662	76	LBL	702	48	EXC
623	75	-	663	10	E*	703	43	RCL
624	53	(664	98	ADV	704	10	10
625	43	RCL	665	43	RCL	705	67	EQ
626	11	11	666	19	19	706	16	A*
627	75	-	667	99	PRT	707	61	GTD
628	43	RCL	668	91	R/S	708	17	B*
629	12	12	669	76	LBL	709	76	LBL
630	54)	670	15	E	710	49	PRD
631	33	X²	671	87	IFF	711	43	RCL
632	35	1/X	672	00	00	712	10	10
633	54)	673	60	DEG	713	67	EQ
634	95	=	674	81	RST	714	16	A*
635	95	+	675	76	LBL	715	43	RCL
636	43	RCL	676	60	DEG	716	12	12
637	07	07	677	22	INV	717	67	EQ
638	65	X	678	86	STF	718	22	INV
639	43	RCL	679	00	00	719	61	GTD

PROGRAM 2 continued

LABEL ADDRESSES

720	17	B*	760	43	RCL	005	18	C*
721	76	LBL	761	13	13	011	11	A
722	50	IXI	762	22	INV	032	12	B
723	43	RCL	763	67	EQ	037	13	C
724	10	10	764	59	INT	044	14	D
725	22	INV	765	04	4	049	16	A*
726	86	STF	766	44	SUM	067	28	LOG
727	40	IND	767	07	07	095	29	CP
728	00	00	768	76	LBL	107	17	B*
729	22	INV	769	59	INT	113	39	CDS
730	67	EQ	770	86	STF	122	38	SIN
731	57	ENG	771	40	IND	141	30	TAN
732	01	1	772	07	07	166	36	PGM
733	42	STD	773	87	IFF	197	37	P/R
734	07	07	774	00	00	202	47	CMS
735	76	LBL	775	17	B*	214	22	INV
736	57	ENG	776	87	IFF	285	23	LNK
737	43	RCL	777	03	03	385	24	CE
738	12	12	778	16	A*	494	25	CLR
739	22	INV	779	87	IFF	664	10	E*
740	67	EQ	780	04	04	671	15	E
741	58	FIX	781	25	CLR	677	60	DEG
742	02	2	782	87	IFF	700	48	EXC
743	44	SUM	783	05	05	708	49	PRD
744	07	07	784	24	CE	720	50	IXI
745	86	STF	785	87	IFF	734	57	ENG
746	40	IND	786	06	06	754	58	FIX
747	07	07	787	23	LNK	767	59	INT
748	87	IFF	788	91	R/S			
749	03	03						
750	59	INT						
751	22	INV						
752	86	STF						
753	40	IND						
754	07	07						
755	76	LBL						
756	58	FIX						
757	43	RCL						
758	12	12						
759	32	XIT						

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